Нелокальное квазипотенциальное уравнение в терминах запаздывающих функций

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(Р е з ю м е)

В работе рассматривается вариант квазипотенциального подхода в задаче двух тел в квантовой электродинамике, сформулированный в терминах запаздывающих функций гейзенберговских полевых операторов и представляющий определенный тип т. н. „нового метода Тамма-Данкова". Показано, что квазипотенциал не содержит расходимостей. Из квазипотенциального уравнения выведено приближенное уравнение, совпадающее с уравнением Брейта для двух взаимодействующих частиц.

Infinite Set of Conservation Laws in the Quantum Sine-Gordon and the Massive Thirring Models

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The infinite set of conserved currents in the Sine-Gordon and the massive Thirring field-theory models is considered. In the quantum theory certain anomalous terms in the Ward identities arise as a result of renormalization. It is shown that the former lead actually only to a slight modification of the classical expressions for the conserved currents. Thus the most remarkable features of the models: particle number conservation, trivial scattering and factorization of the S-matrix are proved in perturbation theory.

1. Introduction

In the last few years some exact solvable nonlinear evolution equations [1] attracted extreme interest in particle physics since they exhibit nontrivial "extended particle" solutions (solitons), which may provide a fundamental basis for constructing unified theories where hadrons arise as coherent excitations of lepton fields [2]. In this sense two dimensional models are known to be of great significance in elucidating the phenomenon of solitons. The most interesting examples are the Sine-Gordon (SG) and the massive Thirring (MTh) models with Lagrangians, respectively,

$$\mathcal{L}^{\mathrm{SG}}=\frac{1}{2}(\partial_\mu\varphi)^2+\frac{m^2}{\beta^2}(\cos\beta\varphi-1),\quad \mathcal{L}^{\mathrm{MTh}}=\frac{i}{2}\bar{\psi}\gamma^\mu\overleftrightarrow{\partial}_\mu\psi-m\bar{\psi}\psi-\frac{\lambda}{4}(\bar{\psi}\gamma^\mu\psi)^2.$$

At the classical level they were exactly solved by the inverse scattering method [3, 4] and were shown to be completely integrable Hamiltonian systems. i. e. they posses an infinite number of conservation laws. The conserved charges have local densities which acquire a particularly simple form with the variables $\tau=\frac{t+x}{2}$, $\sigma=\frac{t-x}{2}\cdot$ In the SG theory the currents may be evaluated from the following recurrence relations:

$$j_\tau^{(n+1)}=\varphi_{,\tau}\partial_\tau\left(\frac{j_\tau^{(n)}}{\varphi_{,\tau}}\right)+\frac{\beta^2}{4}\sum_{k+l=n} j_\tau^{(l)}j_\tau^{(k)}+\varphi_{,\tau}^2\delta_{n,0};\; j_\tau^{(n)}=0,\; n\leq 0,$$

$$j_\sigma^{(n+1)}=\frac{m^2}{\beta}\left(\frac{j_\tau^{(n)}}{\varphi_{,\tau}}\right)\sin\beta\varphi-\frac{2m^2}{\beta^2}\delta_{n,0}\cos\beta\varphi,\; n\geq 0,$$

$$j_\tau^{(n)}=j_0^{(n)}+j_1^{(n)},\quad j_\sigma^{(n)}=j_0^{(n)}-j_1^{(n)};\quad \varphi_{,\tau}\equiv\partial_\tau\varphi\equiv\frac{\partial\varphi}{\partial\tau}.$$

The remaining part of the recurrence relations is obtained from (1.1) by interchanging $\sigma\leftrightarrows\tau$. $j_\tau^{(2k)}$, $k=1, 2, \ldots$ are the total τ-derivatives. $j_\tau^{(2k+1)}$ are homogeneous functions of $2k+2$ degree in ∂_τ and a sum of monomials in $\varphi_{,\tau}$ and its derivatives of $2, 4, \ldots, 2k+2$ degree. The $j_{\tau\sigma}^{(n)}$ obey the conservation law

$$\partial_\sigma j_\tau^{(n)}+\partial_\tau j_\sigma^{(n)}=0.$$

We consider the MTh model (classically) in the case where the ψ's belong to a Grassmann algebra with involution unlike the c-number case [4]. This is reasonable if one has in mind further quantization. Here also an infinite set of conservation laws was found [5]:

$$(1.2)\qquad \partial_\tau(\psi_1^* b_n-h.c.)=im\partial_\sigma(\psi_2^* b_{n-1}+h.c.),$$

where b_n are recursively determined from

$$(1.3)\qquad b_{n+1}=\partial_\sigma b_n-i\lambda\psi_1^*\psi_1 b_n-2i\lambda\sum_{\substack{k+l=n\\ k\neq 0}} b_k^*\psi_1 b_l+\psi_1\delta_{n+1,0};\; b_n=0,\; n<0.$$

The remaining part of the conserved currents is obtained from (1.2) by means of the changes $1\rightleftarrows 2$, $\sigma\rightleftarrows\tau$. All currents for even n are total derivatives.

In quantum theory we proceed within the framework of renormalized perturbation theory. The SG is superrenormalizable: the single "point-loop" divergence is removed by normal ordering. In order to set the pole of the full propagator at point $p^2=m^2$ we have to add a finite counterterm of mass renormalization $\frac{1}{2}\Delta m^2(\beta):\varphi^2:$. However, the conservation of $j_\mu^{(n)}$ is thus spoiled. Therefore, following [6] we add a finite counterterm $\frac{1}{2}\delta m^2(\beta):(\cos\beta\varphi-1):$ so as to retain the structure of the $(\cos\beta\varphi-1)_2$ interaction, where the series $\delta m^2(\beta)$ in β is constructed recursively in perturbation theory using the requirement imposed on the pole of the full propagator. Thus we have the following effective Lagrangian:

$$\mathcal{L}=\frac{1}{2}:[(\partial_\mu\varphi)^2-m^2\varphi^2]:+:\left[\frac{\widetilde{m}^2(\beta)}{\beta^2}(\cos\beta\varphi-1)+\frac{m^2\varphi^2}{2}\right]:,\;\widetilde{m}^2(\beta)=m^2+\delta m^2(\beta).$$

All $j_\mu^{(n)}(x)$ are composite fields and we shall quantize them according to the Zimmermann normal-product formalism [7] with minimal number of subtractions $\omega(\gamma)=d-2n_0$, where d is the cannonical dimension of the composite operator coinciding with the number of derivatives contained in it, and n_0 is the number of ordinary SG interaction vertices of the subgraph γ containing the composite operator vertex. It was observed [8] that renormalization effects may lead to anomalies in the Ward identities (WI) for $j_\mu^{(n)}(x)$.

Similarly, our discussion of the MTh model is based on the effective Lagrangian [9]:

$$\mathscr{L}=\frac{i}{2}(1+b):\bar{\psi}\gamma^\mu \overleftrightarrow{\partial}_\mu\psi:-(m+a):\bar{\psi}\psi:-\frac{\lambda+c}{4}:(\bar{\psi}\gamma^\mu\psi)^2:,$$

where a, b, c denote finite mass-, field- and charge-renormalization constants, respectively. The classical currents $I_\mu^{(n)}(I_\sigma^{(n)}=\overset{*}{\psi}_1 b_n-h.\text{c.}, \text{etc.})$ are also quantized by normal products with minimal number of subtractions $\omega(\gamma)=d-N(\gamma)/2$, where d is the cannonical composite-operator dimension and $N(\gamma)$ is the number of external fermion lines in the subgraph γ. Here also the normal product renormalization causes the appearance of anomalous terms in the WI for the quantized currents.

The main attempt in this paper is to show that the application of the quantum equations of motion (QEM) [10] and the Zimmermann identities (ZI) relating the normal products with the different number and order of subtractions [7] all anomalous terms of WI in both SG and MTh (see Sect. 2 and 3), leads simply to the addition of certain terms to the classical expressions of $j_\mu^{(k)}$ and $I_\mu^{(n)}$, respectively. The quantized currents thus modified are conserved which in the momentum space on the mass shell is equivalent to the following relations:

$$(1.4)\quad \sum_{j=1}^{m}(p_{j\tau}^{\text{in}})^{2n+1}=\sum_{j=m+1}^{k}(p_{j\tau}^{\text{out}})^{2n+1},\ \sum_{j=1}^{m}(p_{j\sigma}^{\text{in}})^{2n+1}=\sum_{j=m+1}^{k}(p_{j\sigma}^{\text{out}})^{2n+1};\ n=0,1,\dots$$

in any scattering processes described by the matrix element $\langle p_{n+1},\dots p_k;\ \text{out}\,|\,p_1,\dots,p_m;\text{in}\rangle$. It is an observation due to Faddeev and Polyakov [11] that (1.4) is equivalent to the statement that the S-matrix is a pure phase, i. e. the particle number is conserved and the scattering is trivial (the sets of in- and outgoing particle momenta coincide). Besides that, according to ref. [12], the S-matrix is factorized.

The results in Sect. 2 and 3 were reported in a brief form elsewhere [13, 5].

2. Quantum Sine-Gordon Model

In order to derive the WI for $j_\mu^{(n)}$ we shall use the QEM written in terms of Green functions:

$$(2.1)\quad \begin{aligned}\langle TN[B\partial_x^n(\varphi_{,\sigma\tau})](x)X\rangle &= -m^2\langle TN[B\{\partial_x^n\varphi\}](x)X\rangle\\ &+\langle TN\left[B\partial_x^n\left(m^2\varphi-\frac{\widetilde{m}^2}{\beta}\sin\beta\varphi\right)\right](x)X\rangle-i\sum_{j=1}^{k}(\partial_x^n\delta(x-y_j))\langle TN[B](x)\hat{X}^j\rangle;\end{aligned}$$

$$X=\prod_{i=1}^{k}\varphi(y_i),\ \hat{X}^j=\prod_{i\neq j}^{k}\varphi(y_i),$$

where B is an arbitrary monomial in φ and its derivatives, N denotes a normal product with minimal number of subtractions, the curly brackets $\{Q\}$

represent 2 oversubtractions in every subgraph, containing lines correspond to the composite operator Q. The last term in (2.1) is that usually called "contact term" or "covariant Schwinger term" [10].

Let us consider first the case of $j_\mu^{(3)}$ for simplicity $\left(j_\tau^{(3)}=\varphi_{,\tau}\varphi_{,\tau\tau\tau}+\frac{\beta^2}{4}\varphi_{,\tau}^4,\ j_\sigma^{(3)}=\frac{m^2}{\beta}\varphi_{,\tau\tau}\sin\beta\varphi\right)$. Applying (2.1) we get the WI for $j_\mu^{(3)}$:

$$\partial_\sigma\langle TN[j_\tau^{(3)}](x)X\rangle+\partial_\tau\langle TN[j_\sigma^{(3)}](x)X\rangle$$

$$(2.2)\qquad =-i\sum_{j=1}^{k}(\partial_\tau^2\delta(x-y_j))\langle T\varphi_{,\tau}\hat{X}^j\rangle-i\sum_{j=1}^{k}\delta(x-y_j)\langle TN[\beta^2\varphi_{,\tau}^3+\varphi_{,\tau\tau\tau}](x)\hat{X}^j\rangle$$

$$+m^2\langle TN[\varphi_{,\tau\tau\tau}(\varphi-\{\varphi\})+\varphi_{,\tau}(\varphi_{,\tau\tau}-\{\varphi_{,\tau\tau}\})](x)X\rangle+\beta^2m^2\langle TN[\varphi_{,\tau}^3(\varphi-\{\varphi\})](x)X\rangle.$$

The last two terms are "anomalous". The first of them may be transformed into $m^2\langle TN[\partial_\tau(\varphi\varphi_{,\tau\tau}-\{\varphi\varphi_{,\tau\tau}\})]X\rangle$ and added to $j_\sigma^{(3)}$. Then we have to compute the second "anomalous" term by means of the ZI [7]. All graphs contributing in our case are depicted in Fig. 1.

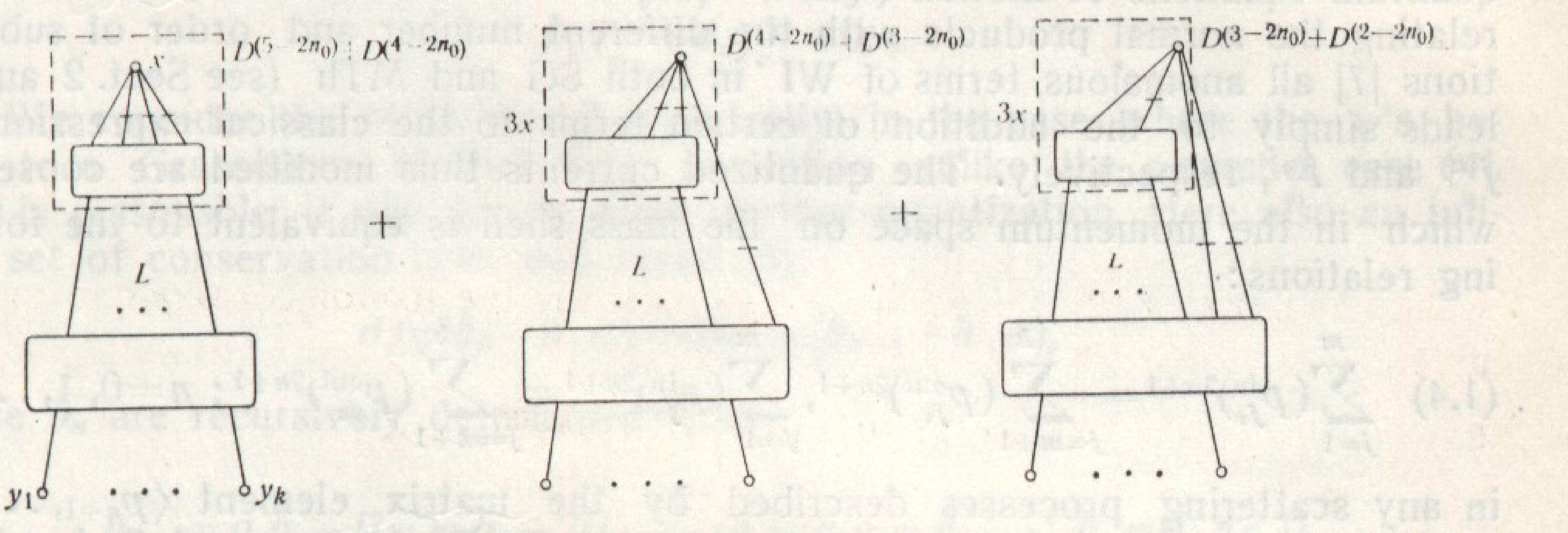

Fig. 1. General graphical form of the Zimmermann identity for $\langle TN[\varphi_{,\tau}^3(\varphi-\{\varphi\})](x)X\rangle$

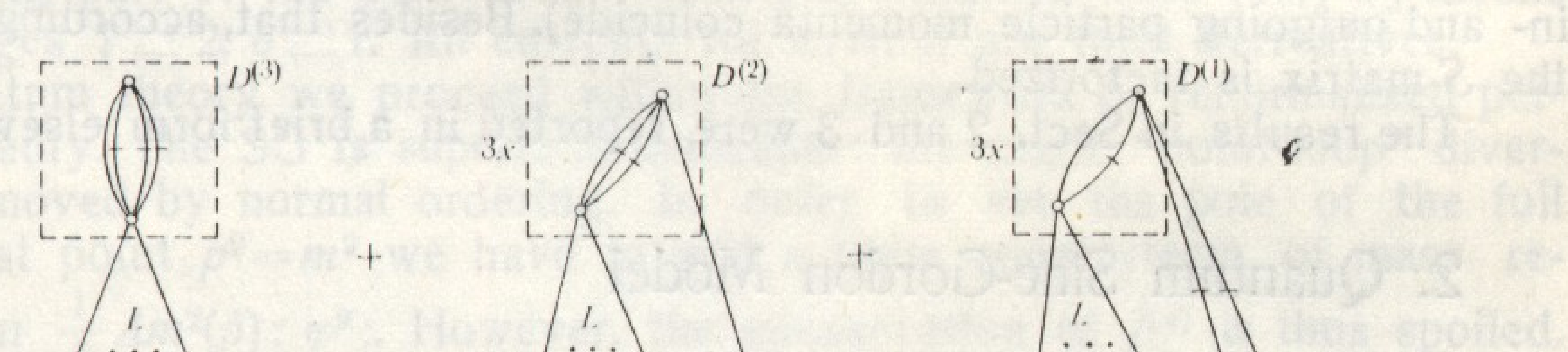

Fig. 2. Graphs contributing to the Zimmermann identity for $\langle TN[\varphi_{,\tau}^3(\varphi-\{\varphi\})](x)X\rangle$

Lines carrying derivatives are marked by a bar. $D^{(s)}$ denotes the $s+1$ — order term of the Taylor expansion operator $\sum'_{i_1 \ldots i_s} p_{i_1}^{\mu_1}\ldots p_{i_s}^{\mu_s}\left(\frac{\partial^s(\ldots)}{\partial p_{i_1}^{\mu_1}\ldots\partial p_{i_s}^{\mu_s}}\right)\Bigg|_{p_i=0}$ acting on the set L of external lines of the subgraphs in the boxes, n_0 is the number of ordinary SG vertices in the latter (see Sect. 1). Thus the contributions of every box represent constants (eventually dependent on β and m^2) in coordinate space which are invariant Lorentz tensors (formed by $g_{\mu\nu}$, $\varepsilon_{\mu\nu}$) times differential operators of order s acting on the lines of L. Two obvious simplifications can be performed: the second $D^{(s)}$ operator vanishes for every box, because of the absence of invariant tensors of odd rank and only boxes with $n_0=1$ contribute as a corollary of the antidiagonal representation of $g_{\mu\nu}$ in σ, τ-components (see Fig. 2).

Analytically the ZI reads (we suppress irrelevant factors):

$$N[\varphi^{(3)}_{,\tau}(\varphi-\{\varphi\})]=c_1\frac{\widetilde{m}^2}{m^2}\beta^2 N[\partial^{(3)}_{\tau}(\cos\beta\varphi)]+\frac{3}{2}c_2\frac{\widetilde{m}^2}{m^2}\beta^2 N[\partial_{\tau}(\varphi^2_{\tau}\cos\beta\varphi)]$$

$$\tag{2.3} +3\frac{\widetilde{m}^2}{m^2}\beta\left(c_3-\frac{1}{2}\beta^2 c_2\right)N[\varphi^3_{,\tau}\sin\beta\varphi],$$

where c_1/m^2, c_2/m^2, c_3/m^2 are the corresponding contributions of all boxes in Fig. 2. Applying QEM (2.1) to the last term in (2.3) we get

$$N[\varphi^3_{,\tau}(\varphi-\{\varphi\})]\left(1-3\beta^2\left(c_3-\frac{1}{2}\beta^2 c_2\right)\right)=c_1\frac{\widetilde{m}^2}{m^2}\beta^2 N[\partial^3_{\tau}(\cos\beta\varphi)]$$

$$\tag{2.4} +\frac{3}{2}c^2\frac{\widetilde{m}^2}{m^2}\beta^2 N[\partial_{\tau}(\varphi^2_{,\tau}\cos\beta\varphi)]+\frac{3}{4}\frac{\widetilde{m}^2\beta^2}{m^2}\left(c_3-\frac{1}{2}c_2\beta^2\right)N[\partial_{\sigma}(\varphi^4_{,\tau})]$$

$$-3i\frac{\beta^2\widetilde{m}^2}{m^2}\left(c_3-\frac{1}{2}\beta^2 c_2\right)\sum_{j=1}^{k}\delta(x-y_j)N[\varphi^{3^{\cdot}}_{,\tau}]\frac{\delta}{\delta\varphi(y_j)}.$$

Inserting (2.4) in (2.2) we obtain a new WI for the modified current $\widetilde{j}^{(3)}_{\mu}$ without anomalies:

$$\tag{2.5} \partial_{\sigma}\langle TN[\widetilde{j}^{(3)}_{\tau}](x)X\rangle+\partial_{\tau}\langle TN[\widetilde{j}^{(3)}_{\sigma}](x)X\rangle$$

$$=-i\sum_{j=1}^{k}{}'\delta(x-y_j)\langle TN\left[2\varphi_{,\tau\tau\tau}+\beta^2\varphi^3_{,\tau}\left(1-3\beta^2\left(c_3-\frac{1}{2}\beta^2 c_2\right)\right)^{-1}\right](y_j)\hat{X}^j;$$

$$\widetilde{j}^{(3)}_{\tau}(x)=N\left[\varphi_{,\tau}\varphi_{,\tau\tau\tau}+\frac{\beta^2}{4}\left(1-\frac{3\beta^2\left(c_3-\frac{1}{2}\beta^2 c_2\right)}{1-3\beta^2\left(c_3-\frac{1}{2}\beta^2 c_2\right)}\right)\varphi^4_{,\tau}\right],$$

$$\widetilde{j}^{(3)}_{\sigma}(x)=N\left[\varphi_{,\tau\tau}\sin\beta\varphi\,.\,(1+c_1\beta^4+c_3\beta^2)\frac{\widetilde{m}^2}{\beta}+\widetilde{m}^2\beta^2\left(c_3-\frac{3}{2}c_2\beta^2+c_1\beta^4\right)\varphi^2_{,\tau}\cos\beta\varphi\right].$$

It is obvious from the preceding analysis that the only source of "anomalies" is contained in terms like $\langle TN[Q\partial^n_x(\varphi-\{\varphi\})](x)X\rangle$ where Q is a certain monomial in φ and its derivatives. In the general case we have from (1.1)

$$\tag{2.6} j^{(2n+1)}_{\tau}(x)=\sum_{r=0}^{n}\beta^{2r}\sum_{\substack{r_1,\dots,r_s,\\ s\geq 0}}c^s_{r_1\dots r_s}(\partial_{\tau}\varphi)^{r_1}\dots(\partial_{\tau}\varphi)^{r_s}\sum_{l=1}^{s}r_l=2r+2,\sum_{l=1}^{s}lr_l=2n+2,$$

where $c^s_{r_1\dots r_s}$ are real numbers (nonnegative). Proceeding in the same way as in the $j^{(3)}_{\mu}$-case we obtain the following "anomalous" terms in the WI for $j^{(2n+1)}_{\mu}(x)$:

$$(2.7)\quad m^2\sum_{r=0}^{n}\beta^{2r}\sum_{\substack{r_1\dots r_s,\\ s\geq 0,\dots}} c^s_{r_1\dots r_s}\sum_{l=1}^{s} N[(\partial_\tau\varphi)^{r_1}\dots r_l(\partial^l_\tau\varphi)^{r_l-1}\partial_\tau^{l-1}(\varphi-\{\varphi\}\dots(\partial^s_\tau\varphi)^{r_s}]$$

$$=m^2N[\partial_\tau Q^{(2n+1)}(\varphi)]+m^2\sum_{l=1}^{n'}N[B_l^{(2n+1)}(\varphi_{,\tau})(\varphi-\{\varphi\})].$$

Here $Q^{(2n+1)}(\varphi)$, $B_l^{(2\ +1)}(\varphi_{,\tau})$ are monomials in φ and its derivatives, $B_l^{(2n+1)}(\varphi_{,\tau})$, have the same general form like (2.6) but with $\sum_{l=1}^{s} r_l=2r+1$, $\sum_{l=1}^{s} lr_l=2n+1$. Applying the ZI to the second term in (2.7) we obtain

$$(2.8)\quad \begin{aligned} N[B_l^{(2n+1)}(\varphi_{,\tau})(\varphi-\{\varphi\})] &= \frac{\widetilde{m}^2}{m^2}\alpha_l(\beta)N[\partial_\tau P_l^{(2n+1)}(\varphi_{,\tau})] \\ &+\frac{\widetilde{m}^2}{m^2}\sum_{p=1}^{n'}\gamma_{lp}(\beta)N[B_p^{(2n+1)}(\varphi_{,\tau})\sin\beta\varphi], \end{aligned}$$

where $P_l^{(2n+1)}(\varphi_{,\tau})$ are other monomials, $\alpha_l(\beta)$ and $\gamma_{lp}(\beta)$ are **finite** series in β with zero constant term, the coefficients of which may be computed explicitly. On the other hand, the QEM give

$$(2.9)\quad \begin{aligned} \frac{\widetilde{m}^2}{\beta}N[B_l^{(2n+1)}(\varphi_{,\tau})\sin\beta\varphi] &= N[\partial_\sigma R_l^{(2n+1)}(\varphi_{,\tau})]-\frac{\widetilde{m}^2}{\beta}\sum_{p=1}^{n'}a_{lp}N[B_p^{(2n+1)}(\varphi_{,\tau})\sin\beta\varphi] \\ &+m^2\sum_{p=1}^{n'}b_{lp}N[B_p^{(2n+1)}(\varphi-\{\varphi\})]-i\sum_{j=1}^{k}\sum_{p=1}^{n'}c_{lp}N[B_p^{(2n+1)}]\delta(x-y_j)\frac{\delta}{\delta\varphi(y_j)}, \end{aligned}$$

where $R_l^{(2n+1)}(\varphi_{,\tau})$ are monomials, a_{lp}, b_{lp}, c_{lp} are nonnegative integers, (2.8) and (2.9) are a nonhomogeneous set of linear algebraic equations for the "anomalous" terms with nonvanishing determinant (at least in perturbation theory) and with Schwinger terms and total derivatives of field monomials as nonhomogeneous terms. Its solution reads:

$$\begin{aligned} m^2N[B_l^{(2n+1)}(\varphi_{,\tau})(\varphi-\{\varphi\})] &= \sum_{s=1}^{n'}\Omega_{ls}^{-1}(\beta)N\Big[\widetilde{m}^2\alpha_s(\beta)\partial_\tau P_s^{(2n+1)}(\varphi_{,\tau})+\beta\sum_{p,q=1}^{n'}\gamma_{sp}(\beta)\omega^{-1}_{pq} \\ &\times\Big(\partial_\sigma R_q^{(2n+1)}(\varphi_{,\tau})-i\sum_{j=1}^{k}\sum_{r=1}^{n'}c_{qr}B_r^{(2n+1)}(\varphi_{,\tau})\delta(x-y_j)\frac{\delta}{\delta\varphi(y_j)}\Big)\Big]; \end{aligned}$$

$$\omega_{lp}=\delta_{lp}+a_{lp},\quad \Omega_l^r(\beta)=\delta_{lr}-\beta\sum_{p,q=1}^{n'}\gamma_{lp}(\beta)\omega^{-1}_{pq}b_{qr}.$$

In this way we obtain new WI for the modified quantum currents $\widetilde{j}^{(2n+1)}_\mu$:

$$\partial_\sigma\langle TN[\widetilde{j}^{(2n+1)}_\tau](x)X\rangle+\partial_\tau\langle TN[\widetilde{j}^{(2n+1)}_\sigma](x)X\rangle=-i\sum_{j=1}^{k}\delta(x-y_j)\sum_{l,r=1}^{n'}\Big(\delta_{lr}$$

$$+\beta\sum_{s,p,q=1}^{n'}\Omega_{ls}^{-1}(\beta)\gamma_{sp}(\beta)\omega_{pq}^{-1}c_{qr}\Big)\langle TN[B_r^{(2n+1)}(\varphi_{,\tau})](y_j)\hat{X}^j\rangle$$

$$\text{(2.10)}\quad \tilde{j}_\tau^{(2n+1)}(x)=N[j_\tau^{(2n+1)}(x)-\beta\sum_{l,s,p,q=1}^{n'}\Omega_{ls}^{-1}(\beta)\gamma_{sp}(\beta)\omega_{pq}^{-1}R_q^{(2n+1)}(\varphi_{,\tau})]$$

$$\tilde{j}_\sigma^{(2n+1)}(x)=N\Big[j_\sigma^{(2n+1)}(x)-\tilde{m}^2(\beta)\sum_{l,s=1}^{n'}\Omega_{ls}^{-1}(\beta)\alpha_s(\beta)P_s^{(2n+1)}(\varphi_{,\tau})-m^2Q^{(2n+1)}(\varphi)\Big].$$

The integrated (over x) (2.10) in momentum space on the mass shell gives

$$\text{(2.11)}\quad \Big(\sum_{l,r=1}^{n'}F_{lr}(\beta)G_r^{(2n+1)}(\beta,m)\Big)\Big(\sum_{j=1}^{m}(p_{j\tau}^{\text{in}})^{2n+1}-\sum_{j=m+1}^{k}(p_{j\tau}^{\text{out}})^{2n+1}\Big)$$
$$\times\langle p_{m+1},\ldots,p_k;\text{ out}\,|\,p_1,\ldots,p_m;\text{ in}\rangle=0,$$

where we have used the following notations:

$$F_{lr}(\beta)=\delta_{lr}+\beta\sum_{s,p,q=1}^{n'}\Omega_{ls}^{-1}(\beta)\gamma_{sp}(\beta)\omega_{pq}^{-1}c_{qr},$$

$$\Phi\langle TN[B_r^{(2n+1)}(\varphi_{,\tau})](y_j)\hat{X}^j\rangle=\Phi\langle TN[B_r^{(2n+1)}(\varphi_{,\tau})](y_j)\varphi\rangle^{\text{prop}}\big|_{p_j^2=m^2}$$

$$\times\langle p_{m+1},\ldots;\text{ out}\,|\,p_1,\ldots,\text{ in}\rangle=\pm(p_{j\tau}^{\substack{\text{in}\\ \text{out}}})^{2n+1}G_r^{2n+1}(\beta,m)\,\langle p_{m+1},\ldots;\text{ out}\,|\,p_1,\ldots;\text{in}\rangle$$

Here Φ is the composition of the following three operations: Fourier transformation, amputation and restriction on the mass shell[1]. (2.11) is the desired result (see (1.4) and the end of Sect. 1)[2].

3. Quantum Massive Thirring Model

In this section the same technique as developed for the SG model is applied. We shall use the standard representation for the γ-matrices:

$$\gamma^0=\begin{pmatrix}0&1\\1&0\end{pmatrix},\quad \gamma^1=\begin{pmatrix}0&-1\\1&0\end{pmatrix},\quad \gamma^5=\gamma^0\gamma^1=\begin{pmatrix}1&0\\0&-1\end{pmatrix}.$$

The QEM of the massive Thirring model read ($\varrho_\alpha=\psi_\alpha^*\psi_\alpha$, $\alpha=1,2$):

$$\langle TN[B\partial_x^n(\psi_{,\tau})](x)X\rangle=-im'\langle TN[B\partial_x^n\psi_2](x)X\rangle$$

$$\text{(3.1)}\quad -i\lambda'\langle TN[B\partial_x^n(\varrho_2\psi_1)](x)X\rangle+\frac{1}{1+b}\sum_{j=1}^{k}(\partial_x^n\delta(x-y_j))\langle TN\Big[B\frac{\delta}{\delta\psi_1^*(y_j)}\Big]X\rangle$$
$$+\frac{im}{1-b}\langle TN[B\partial_x^n(\psi_2-\{\psi_2\})](x)X\rangle;$$

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1 The superscript "prop" denotes amputated, one-particle irreducible Green functions.

2 (1.4) in the SG model was established in [6] using unitarity and imposing some requirements on the analyticity of the scattering amplitude.

$$\lambda'=\frac{\lambda+c}{1+b},\ m'=\frac{m+a}{1+b};\ X=\prod_{i=1}^{k}\psi(x_i)\prod_{i=1}^{k}\bar{\psi}(y_i);\ \psi_{1,\tau}\equiv\partial_\tau\psi_1,\text{etc.},$$

where $\{\ldots\}$ denotes 1 oversubtraction. The rest of the QEM is obtained from (3.1) by means of the changes $1\leftrightarrows 2$, $\sigma\leftrightarrows\tau$ and/or conjugation of the composite operators. Now the change $\lambda\rightarrow\lambda'$ is performed in the reccurrence relations (1.3). Using the latter and (3.1) we obtain the following identity (further on we suppress all irrelevant factors):

$$\partial_\tau b_{n+1}=-m'^2 b_n-i\lambda'\varrho_2 b_{n+1}+2m'\lambda'\sum_{k+l=n}b_k^*\psi_2 b_l+A_{n+1}+S.t.:\ n\geq 0, \tag{3.2}$$

where "$S.t.$" stands for the omitted Schwinger terms which are of no importance in our consideration. One can easily verify by induction in n the following recurrence relation for A_n:

$$\begin{aligned}A_{n+1}=\partial_\sigma A_n-i\lambda'\varrho_1 A_n-2i\lambda'\sum_{\substack{k+l=n\\k,l\neq 0}}(A_k^*\psi_1 b_l+b_k^*\psi_1 A_l)\\+2\lambda'm'\frac{im}{1+b}\sum_{k+l=n-1}b_k^*(\psi_1-\{\psi_1\})b_l-2i\lambda'\frac{im}{1+b}\sum_{\substack{k+l=n\\k,l\neq 0}}b_k^*(\psi_2-\{\psi_2\})b_l\\-i\lambda'\frac{im}{1+b}(\{\bar{\psi}\}\psi-\bar{\psi}\{\psi\})b_n-im'\frac{im}{1+b}(\psi_1-\{\psi_1\})\delta_{n,0};\ n\geq 0,\\A_0=\frac{im}{1+b}(\psi_2-\{\psi_2\}).\end{aligned} \tag{3.3}$$

Using (3.1), (3.2) and (3.3), the WI for the conserved currents acquire the form

$$\begin{aligned}\partial_\tau(\psi_1^* b_n-h.c.)-im'\partial_\sigma(\psi_2^* b_{n-1}+h.c.)=\psi_1^*\partial_\sigma A_{n-1}\\+2i\lambda'\varrho_1\sum_{\substack{k+l=n-1\\k,l\neq 0}}(A_k^* b_l+b_k^* A_l)-im'\frac{im}{1+b}(b_{n-1}^*-\partial_\sigma b_{n-2}^*)(\psi_1\{\psi_1\})-\\-\frac{im}{1+b}(b_n^*\partial_\sigma b_{n-1}^*-i\lambda'\varrho_1 b_{n-1}^*)(\psi_2-\{\psi_2\})-i\lambda'\frac{im}{1+b}(\{\bar{\psi}\}\psi-\bar{\psi}\{\psi\})\psi_1^* b_{n-1}\\-\frac{im}{1+b}(\psi_2^*-\{\psi_2^*\})b_n+im'\frac{im}{1+b}(\psi_1^*-\{\psi_1^*\})b_{n-1}-h.c.+S.t.\end{aligned} \tag{3.4}$$

Here "$S.t.$" again stands for the omitted Schwinger terms which in momentum space on the mass shell give contributions of the type (2.11). They will be suppressed in the sequel. All I_b^n have covariant structure of the $\sigma\ldots\sigma$ ($n+1$ times)-component of a $O(1,1)$ (Lorentz) tensor of rank $n+1$, I_τ^n is the $\sigma\ldots\sigma$ ($n-1$ times)-Lorentz tensor component. Hence all "anomalous" terms (the right hand side of (3.4)) have the covariant structure of a $\sigma\ldots\sigma$ (n times)-Lorentz tensor component. It is obvious that (in complete analogy with the SG case) only the last term in (3.1) and its counterparts are sources of "anomalies". Thus the "anomalous" terms in (3.4) have the following general form

$$\partial_\sigma C_n+\frac{im}{1+b}\sum_j P_j^n(\overline{\psi\psi}\underbrace{{}_{,\sigma\ldots\sigma}}_{j}-\overline{\{\psi\}\psi}\underbrace{{}_{,\sigma\ldots\sigma}}_{j}) \tag{3.5}$$

$$+\frac{im}{1+b}\sum_j Q_j^n(\bar{\psi}\gamma_\sigma\psi_{,\underbrace{\sigma\ldots\sigma}_{j}}-\{\bar{\psi}\}\gamma_\sigma\psi_{,\underbrace{\sigma\ldots\sigma}_{j}})-h.c.,\quad \gamma_\sigma=\gamma_0-\gamma_1.$$

Here C_n denote Lorentz covariant polynomials in ψ and $\bar{\psi}$ of canonical dimension n, $P_j^{(n)}$ and $Q_j^{(n)}$ are Lorentz covariant polynomials in ψ and $\bar{\psi}$ with coefficients eventually dependent on powers of $i\lambda'$ and im'.

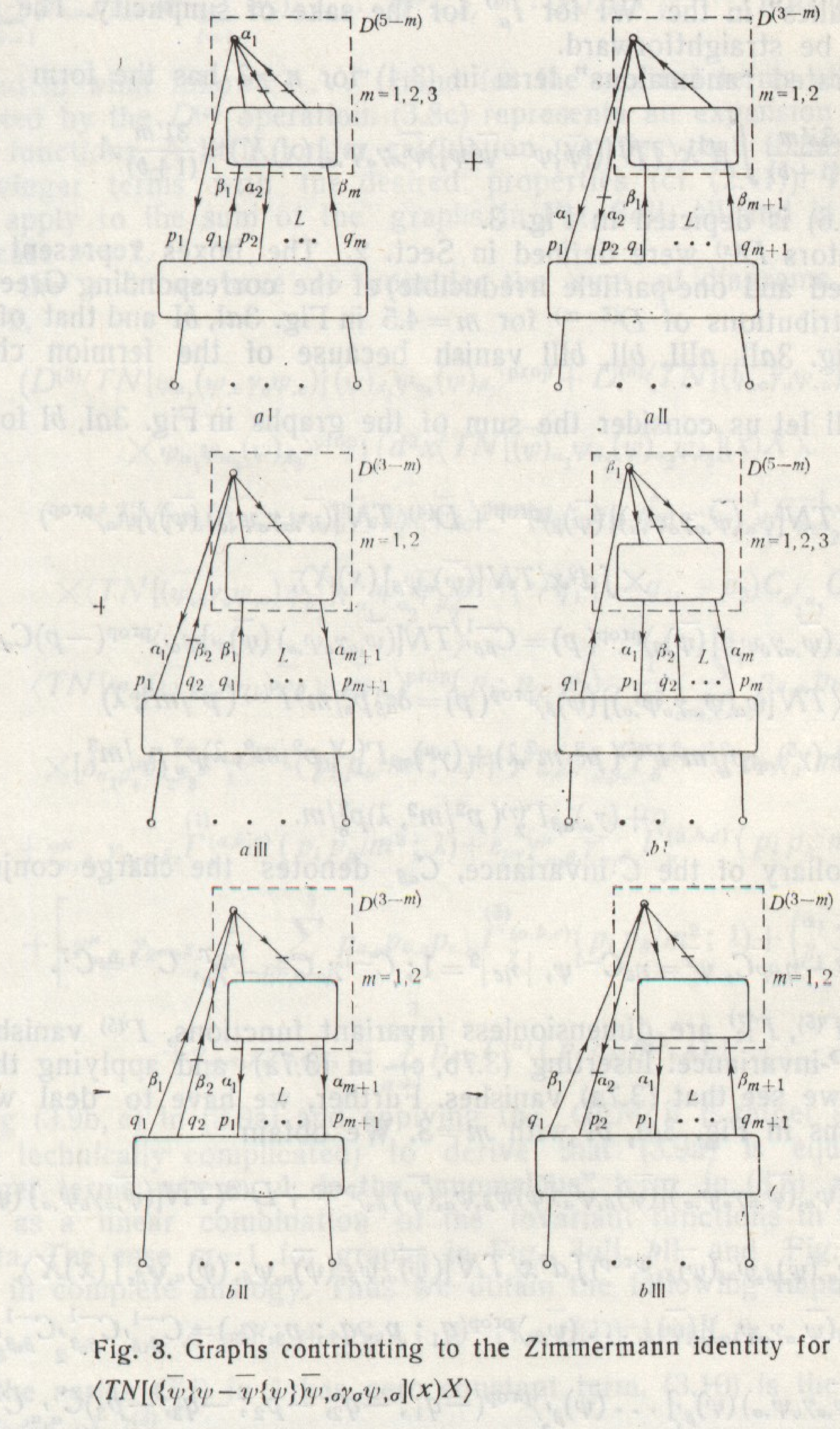

Fig. 3. Graphs contributing to the Zimmermann identity for $(TN[(\{\bar{\psi}\}\psi-\bar{\psi}\{\psi\})\bar{\psi}_{,\sigma}\gamma_\sigma\psi_{,\sigma}](x)X)$

It is simpler to deal with the integrated (over x) WI (3.4) in order to avoid all total derivative terms. The analysis of the "anomalies" (3.5) completely resembles the line of argument in Sec. 2, i. e. making use of the ZI for normal products and the QEM we obtain again a nonhomogeneous set of linear algebraic equations for all "anomalous" terms (cf. (2.8), (2.0)). In the same way the latter reduce to Schwinger terms only and consequently (1.4) is satisfied. Of course the ZI in the MTh model are much more complicated, however, the anticommutation of the ψ's under the normal product symbol, Lorentz covariance and C- and P-invariance place severe restrictions on them, thus allowing to complete the analysis. We shall illustrate the technique only for the "anomalies" in the WI for $I_\mu^{(3)}$ for the sake of simplicity. The general case then will be straightforward.

The integrated "anomalous" term in (3.4) for $n=3$ has the form

$$(3.6)\qquad \frac{3\lambda' m}{(1+b)}\int d^2x\langle TN[(\{\bar{\psi}\}\psi-\bar{\psi}\{\psi\})\bar{\psi}_{,\sigma}\gamma_\sigma\psi_{,\sigma}](x)X\rangle=\frac{3\lambda' m}{(1+b)}A$$

The ZI for (3.6) is depicted in Fig. 3.

The operators $D^{(s)}$ were defined in Sect. 2. The boxes represent proper parts (amputated and one-particle irreducible) of the corresponding Green functions. The contributions of $D^{(5-m)}$ for $m=4.5$ in Fig. 3aI, bI and that of $D^{(3-m)}$ for $m=3$ in Fig. 3aII, aIII, bII, bIII vanish because of the fermion character of the fields.

First of all let us consider the sum of the graphs in Fig. 3aI, bI for $m=1$. We have

$$(3.7a)\qquad (D^{(4)}\langle TN[\psi_\alpha(\bar{\psi}_{,\sigma}\gamma_\sigma\psi_{,\sigma})](\bar{\psi})_\beta\rangle^{\text{prop}}+D^{(4)}\langle TN[(\bar{\psi}_{,\sigma}\gamma_\sigma\psi_{,\sigma})(\bar{\psi})_\beta]\psi_\alpha\rangle^{\text{prop}})\times\int d^2x\langle TN[(\bar{\psi})_\alpha\psi_\beta](x)X\rangle,$$

$$(3.7b)\qquad \langle TN[\psi_\alpha(\bar{\psi}_{,\sigma}\gamma_\sigma\psi_\sigma)](\bar{\psi})_\beta\rangle^{\text{prop}}(p)=C^{-1}_{\beta\beta'}\langle TN[(\bar{\psi}_{,\sigma}\gamma_\sigma\psi_{,\sigma})(\bar{\psi})_{\beta'}]\psi_{\alpha'}\rangle^{\text{prop}}(-p)C_{\alpha'\alpha},$$

$$(3.7c)\qquad \begin{aligned}\langle TN[\psi_\alpha(\bar{\psi}_{,\sigma}\gamma_\sigma\psi_{,\sigma})](\bar{\psi})_\beta\rangle^{\text{prop}}(p)&=\delta_{\alpha\beta}p_\sigma^3/m^2\Gamma^{(1)}(p^2/m^2,\lambda)\\&+(\gamma^5)_{\alpha\beta}p_\sigma^3/m^2\Gamma^{(5)}(p^2/m^2,\lambda)+(\gamma^\mu)_{\alpha\beta}\Gamma_1^{(\gamma)}(p^2/m^2,\lambda)p_\sigma^3p_\mu/m^3\\&+(\gamma_\sigma)_{\alpha\beta}\Gamma_2^{(\gamma)}(p^2/m^2,\lambda)p_\sigma^2/m.\end{aligned}$$

(3.7b) is a corollary of the C-invariance, $C_{\alpha\beta}$ denotes the charge conjugation matrix:

$$\psi^c=-\eta_c\bar{\psi}C,\ \bar{\psi^c}=\eta_cC^{-1}\psi,\ |\eta_c|^2=1;\ C^{-1}\gamma_\mu C=-\gamma_\mu^T,\ C^{-1}=C^T.$$

In (3.7c) $\Gamma^{(1)}$, $\Gamma^{(5)}$, $\Gamma_{12}^{(\gamma)}$ are dimensionless invariant functions, $\Gamma^{(5)}$ vanishes because of the P-invariance. Inserting (3.7b, c) in (3.7a) and applying the definition of $D^{(s)}$ we see that (3.7a) vanishes. Further, we have to deal with the sum of diagrams in Fig. 3aI, bI with $m=3$. We obtain

$$(3.8a)\qquad \begin{aligned}&(D^{(2)}\langle TN[\psi_{\alpha_1}(\bar{\psi}_{,\sigma}\gamma_\sigma\psi_{,\sigma})](\bar{\psi})_{\beta_1}\psi_{\alpha_2}(\bar{\psi})_{\beta_2}\psi_{\alpha_3}(\bar{\psi})_{\beta_3}\rangle^{\text{prop}}+D^{(2)}\langle TN[(\bar{\psi}_{,\sigma}\gamma_\sigma\psi_{,\sigma})(\bar{\psi})_{\beta_1}]\\&\times\psi_{\alpha_1}\psi_{\alpha_2}(\bar{\psi})_{\beta_2}\psi_{\alpha_3}(\bar{\psi})_{\beta_3}\rangle^{\text{prop}})\int d^2x\langle TN[(\bar{\psi})_{\alpha_1}\psi_{\beta_1}(\bar{\psi})_{\alpha_2}\psi_{\beta_2}(\bar{\psi})_{\alpha_3}\psi_{\beta_3}](x)X\rangle,\end{aligned}$$

$$(3.8б)\qquad \begin{aligned}&\langle TN[\psi_{\alpha 1}(\bar{\psi}_{,\sigma}\gamma_\sigma\psi_{,\sigma})](\bar{\psi})_{\beta_1}\ldots(\bar{\psi}_{\beta_3})\rangle^{\text{prop}}(q_1;p_2,q_2;p_3,q_3)=C^{-1}_{\beta_1\beta_1'}C^{-1}_{\beta_2\beta_2'}C^{-1}_{\beta_3\beta_3'}\\&\times\langle TN[(\bar{\psi}_{,\sigma}\gamma_\sigma\psi_{,\sigma})(\bar{\psi})_{\beta_1'}]\ldots(\bar{\psi})_{\beta_3'}\rangle^{\text{prop}}(-q_1;-q_2,-p_2;-q_3,-p_3)C_{\alpha_1'\alpha_1}C_{\alpha_2'\alpha_2}C_{\alpha_3'\alpha_3}\end{aligned}$$

$$\langle TN[\psi_{\alpha_1}(\bar{\psi}_{,\sigma}\gamma_\sigma\psi_{,\sigma})](\bar{\psi})_{\beta_1}\ldots(\bar{\psi})_{\beta_3}\rangle^{\text{prop}}(p_1;p_2,p_3;p_4,p_5)=\sum_{a,b=1}^{5}p_{a,\sigma}p_{b,\sigma}/m^3$$

$$
\text{(3.8c)}\quad \sum_{i=1}^{3}[\delta_{\alpha_1\beta_1}\cdots(\gamma)_{\sigma\alpha_i\beta_i}\cdots\delta_{\alpha_3\beta_3}\Gamma_1^{(i)\,(a,b)}(p_e p_k/m^2;\lambda)+(\gamma^5)_{\alpha_1\beta_1}\cdots(\gamma)_{\sigma\alpha_i\beta_i}\cdots(\gamma^5)_{\alpha_3\beta_3}
$$

$$
\times\Gamma_5^{(i)\,(a,b)}(p_l p_k/m^2;\lambda)+(\gamma^\mu)_{\alpha_1\beta_1}\cdots(\gamma_\sigma)_{\alpha_i\beta_i}\cdots(\gamma_\mu)_{\alpha_3\beta_3}\Gamma_\gamma^{(i)\,(a,b)}(p_l p_k/m^2;\lambda)]
$$

$$
+\sum_{a,b=1}^{5} p_{a,\sigma}p_{b,\mu}/m^3\sum_{i=1}^{3}(\gamma_\sigma)_{\alpha_1\beta_1}\cdots(\gamma^\mu)_{\alpha_i\beta_i}\cdots(\gamma_\sigma)_{\alpha_3\beta_3}\Gamma_\gamma^{(i)\,(a\,b)}(p_l p_k/m^2;\lambda)+\cdots
$$

Here and in what follows "…" stand for the omitted terms all of which are annihilated by the $D^{(s)}$ operation. (3.8c) represents an expansion in Lorentz invariant functions. A little longer calculation verifies that (3.8a) reduces only to Schwinger terms with the desired properties (cf. (2.11)). The same arguments apply to the sum of the graphs in Fig. 3*a*II, *b*II and in Fig. 3*a*III, *b*III in the case $m=2$.

At the end we have to consider the sum of diagrams in Fig. 3*a*I, *b*I for $m=2$.

$$
\text{(3.9a)}\quad \begin{aligned}&(D^{(3)}\langle TN[\psi_{\alpha_1}(\bar\psi_{,\sigma}\gamma_\sigma\psi_{,\sigma})](\bar\psi)_{\beta_1}\psi_{\alpha_2}(\bar\psi)_{\beta_2}\rangle^{\text{prop}}+D^{(3)}\langle TN[(\bar\psi_{,\sigma}\gamma_\sigma\psi_{,\sigma})(\bar\psi)_{\beta_1}]\\ &\times\psi_{\alpha_1}\psi_{\alpha_2}(\bar\psi)_{\beta_2}\rangle^{\text{prop}})\int d^2x\langle TN[(\bar\psi)_{\alpha_1}\psi_{\beta_1}(\bar\psi)_{\alpha_2}\psi_{\beta_2}](x)X\rangle,\end{aligned}
$$

$$
\text{(3.9b)}\quad \begin{aligned}&\langle TN[\psi_{\alpha_1}(\bar\psi_{,\sigma}\gamma_\sigma\psi_{,\sigma})]\bar\psi_{\beta_1}\psi_{\alpha_2}\bar\psi_{\beta_2}\rangle^{\text{prop}}(q_1;q_2,q_2)-C^{-1}_{\beta_1\beta_1'}C^{-1}_{\beta_2\beta_2'}\\ &\times\langle TN[(\bar\psi_{,\sigma}\gamma_\sigma\psi_{,\sigma})\psi_{\beta_1'}]\psi_{\alpha_1'}\psi_{\alpha_2'}\bar\psi_{\beta_2'}\rangle^{\text{prop}}(-q_1;-q_2,-p_3)C_{\alpha_1'\alpha_1}C_{\alpha_2'\alpha_2},\end{aligned}
$$

$$
\text{(3.9c)}\quad \begin{aligned}&\langle TN[\psi_{\alpha_1}(\bar\psi_{,\sigma}\gamma_\sigma\psi_{,\sigma})\bar\psi_{\beta_1}\psi_{\alpha_2}\bar\psi_{\beta_2}\rangle^{\text{prop}}(p_1;p_2,p_3)=\frac{1}{m^3}\sum_{a,b,c=1}^{3}p_{a,\sigma}p_{b,\sigma}p_{c,\sigma}\\ &\times[\delta_{\alpha_1\beta_1}\delta_{\alpha_2\beta_2}\Gamma_1^{(a,b,c)}(p_l p_k/m^2;\lambda)+\gamma^5{}_{\alpha_1\beta_1}\gamma^5_{\alpha_2\beta_2}\Gamma_5^{(a,b,c)}(p_l p_k/m^2;\lambda)\\ &+\gamma^\mu_{\alpha_1\beta_1}\gamma_{\mu,\alpha_2\beta_2}\overset{(1)}{\Gamma}{}_\gamma^{(a,b,c)}(p_l p_k/m^2;\lambda)+\varepsilon_{\mu\nu}\gamma^\mu_{\alpha_1\beta_1}\gamma^\nu_{\alpha_2\beta_2}\overset{(2)}{\Gamma}{}_\gamma^{(a,b,c)}(p_l p_k/m^2;\lambda)]\\ &+\left[\gamma^\mu_{\alpha_1\beta_1}\gamma_{\sigma,\alpha_2\beta_2}\frac{1}{m^3}\sum_{a,b,c=1}^{3}p_{a,\sigma}p_{b,\sigma}p_{c,\mu}\overset{(3)}{\Gamma}{}_\gamma^{(a,b,c)}(p_l p_k/m^2;\lambda)+\begin{pmatrix}\alpha_1\rightleftarrows\alpha_2\\ \beta_1\rightleftarrows\beta_2\end{pmatrix}\right]\\ &+\gamma_{\sigma,\sigma_1\beta_1}\gamma_{\sigma,\alpha_2\beta_2}\frac{1}{m}\sum_{a=1}^{3}p_{a,\sigma}\Gamma_\gamma^{(a)}(p_l p_k/m^2,\lambda)+\cdots\end{aligned}
$$

Inserting (3.9b, c) in (3.9a) and applying the *QEM* it is rather straightforward (though technically complicated) to derive that (3.9a) is equal to $F(\lambda)A+$ Schwinger terms, where A is the "anomalous" term in (3.6) and $F(\lambda)$ is expressed as a linear combination of the invariant functions in (3.9c) for zero momenta. The case $m=1$ for graphs in Fig. 3*a*II, *b*II and Fig. 3*a*III, *b*III is treated in complete analogy. Thus we obtain the following important relation:

$$
\text{(3.10)}\qquad A=G(\lambda)A+S.t.;\ A=(1-G(\lambda))^{-1}\times(S.\,t),
$$

where the series $G(\lambda)$ in λ has zero constant term. (3.10) is the desired result in the case of $I_\mu^{(3)}$.

Applying to the general case (3.5) the same technique as described above which looks here much more complicated, one proves the validity of (1.4) as in the Sine-Gordon case.

Note added in Proof

After submission of this work for publication the papers [14] appeared where the same problems were discussed. The results are in agreement with ours.

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Бесконечная система законов сохранения для уравнения Синус-Гордон и массивной модели Тирринга

Е. Р. Нисимов

(Резюме)

В работе рассматривается бесконечная система сохраняющихся токов для уравнения Синус-Гордон и массивной модели Тирринга. В квантовой теории в результате перенормировки возникают аномальные члены в соответствующих тождествах Уорда. Показано, что в действительности эти ано-

малии ведут только к некоторой модификации классических выражений для токов и таким образом доказаны в рамках теории возмущений замечательные свойства моделей: сохранение числа частиц, тривиальное рассеяние и факторизация S-матрицы.

Моделирование характеристик управляющей системы установки для регистрации широких атмосферных ливней

Й. Стаменов, С. Ушев

ИЯИЯЕ, БАН

Периферийная управляющая система Тянь-Шаньской комплексной установки отбирает в рабочей области $R \leq 10$ m с эффективностью не хуже 90% ливней величиной $N_e \geq 4.10^5$ с возрастным параметром $0{,}6 \leq S \leq 1{,}2$. При $N_e \geq 4.10^5$ эффективность в той же области не зависит от величины ливня N_e. Измеренные пространственные распределения потоков мюонов и электронов на больших расстояниях от оси ливня не искажены влиянием примененной системы отбора ливней.

В работах [1, 2] исследованы пространственные распределения потоков высокоэнергетичных мюонов и электронов, а также энергетический спектр мюонов на больших расстояниях от оси в ливнях величиной 10^5—5.10^6, зарегистрированных на Тянь-Шаньской установке [3]. Сочетание центральной [3] и периферийной [1] управляющей системы дало возможность продвинуться вперед в изучении функций пространственного распределения $\varphi_\mu(r)$ потока мюонов с $E_\mu \geq 5$ GeV в интервале расстояний от оси ливня $8 \text{ m} < r < 180$ m. При этом для изучения характеристик мюонного потока на расстояниях $80 < r < 180$ m использовалось только управление от периферийной мастерной системы (ПЕМА).

Периферийная управляющая система состоит из 10 сцинтилляционных счетчиков, площадью 0,26 m² каждый [1], которые размещены попарно в пяти пунктах регистрации [рис. 1]. Скомпанованный таким образом детекторный крест A мобилен в интервале $r = 0$—150 m от центра комплексной установки. Для формирования управляющего сигнала используются только пять сцинтилляционных счетчиков, расположенных в центре и на расстоянии 20 m от него. Элек-

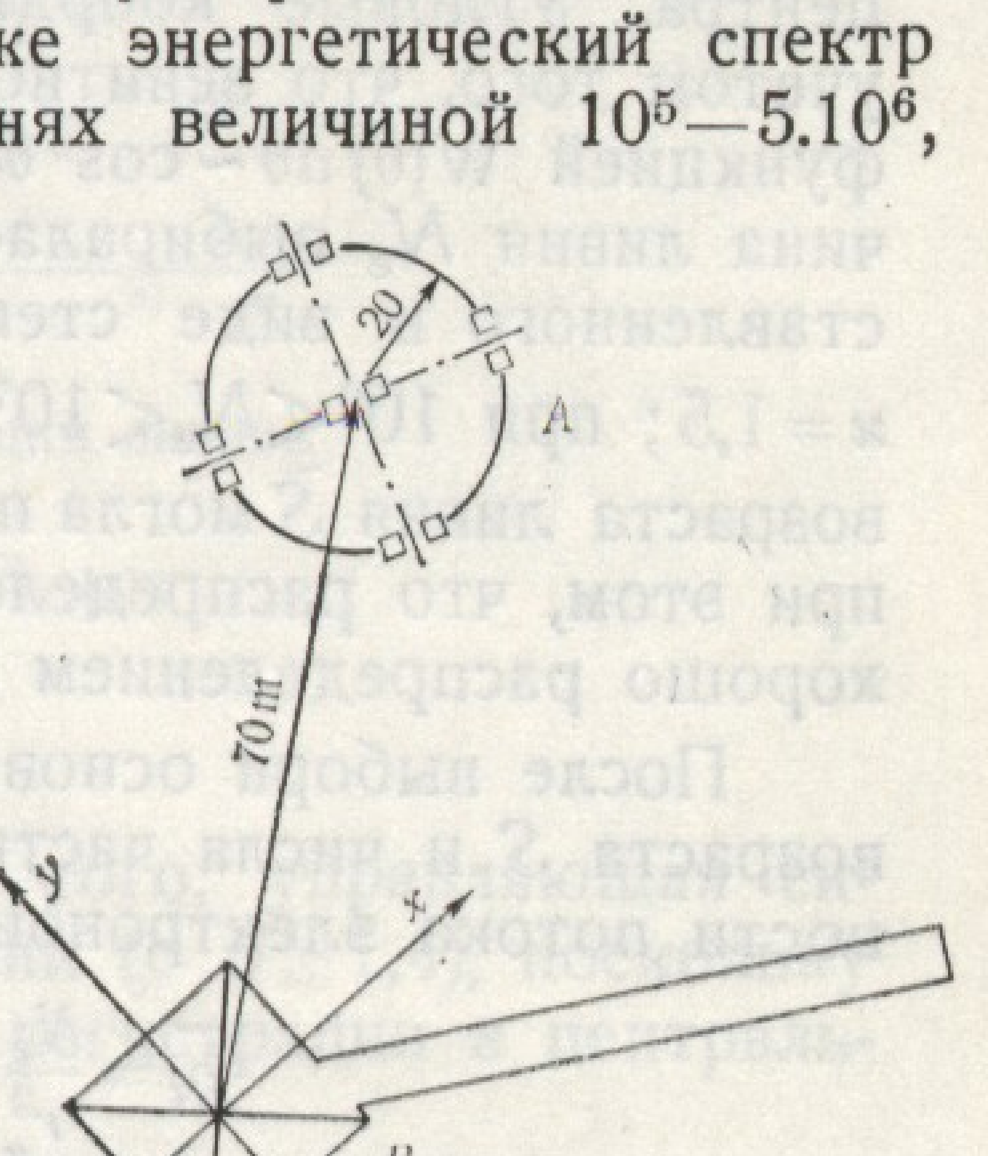

Рис. 1. Геометрия размещения детекторов периферийной управляющей системы:

A — детекторный крест; B — ливневая часть комплексной установки